

A bird's eye view of quantum computers

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ABSTRACT

Quantum computers are discussed in the general framework of computation, the laws of physics and the foundations of quantum mechanics.

1. Introduction

Quantum computation (Benenti & et al. 2004-2007; Nielsen & Chuang 2000) has gained widespread interest as a new interdisciplinary field of research, which benefits from the contributions of physicists, computer scientists, mathematicians, chemists and engineers. A first reason for this interest is that the simulation of quantum many-body systems, a problem of utmost importance in quantum chemistry and biochemistry, is a difficult task for a classical computer as the size of the Hilbert space grows exponentially with the number of particles. On the other hand, these quantum systems can be simulated by means of a quantum computer with resources that grow only polynomially with the system size, see Lloyd (1996) and Lidar and Wang (1999). It is also remarkable that quantum mechanics can help in the solution of basic problems of computer science, such as the prime-factorization problem, for which quantum computation provides an exponential speedup with respect to any known classical computation (Shor 1997) (it is useful to note here that cryptographic schemes extensively used today such as RSA are based on the conjecture that, given a composite odd positive integer, it is “hard” to find its prime factors; hence a large scale quantum computer, if constructed, would break the RSA encryption scheme). Of course, there are problems for which quantum computation leads to an algebraic speedup (for instance, searching a marked item in an unstructured database, see Grover (1997)) or to no speedup at all.

Besides efficiency of computations, an important question is whether the class of problems that can be solved on a quantum computer is the same as on a classical Turing machine. The answer is positive according to the standard model of quantum computation, even though the possibility of a different answer based on infinite-dimensional Hilbert spaces will be briefly mentioned later.

It is convenient to set the quantum computer within the general frame of the technological development of computer machines. Indeed, miniaturization provides us with an intuitive way of understanding why, in the near future, quantum mechanics will become important for computation. The electronics industry for computers grows hand-in-hand with the decrease in size of integrated circuits. This miniaturization is necessary to increase computational power, that is, the number of floating-point operations per second (flops) a computer can perform. Smaller size circuits boost computer power because the communication between components is faster, smaller active components are faster and at the same time their density increases. In the 1950's, electronic computers based on vacuum-tube technology were capable of performing approximately 10^3 floating-point operations per second, while nowadays there exist supercomputers whose power is about 100 teraflops (10^{14} flops). Therefore, today's supercomputers are 10^{11} times more powerful than 50 years ago. It is then clear that it is more appropriate to consider the growth rate of computer power rather than today's computer power. The computational power has been growing at least a factor of two every two years, for the last 50 years. This enormous growth of computational power has been made possible owing to progress in miniaturization, which may be quantified empirically in Moore's law (Moore 1965). This law is the result of a remarkable observation made by Gordon Moore in 1965: the number of transistors on a single integrated-circuit chip doubles approximately every 18 – 24 months. This exponential growth has not yet saturated and Moore's law is still valid. At the present time the limit is approximately 10^8 transistors per chip and the typical size of circuit components is of the order of 100 nanometers. Extrapolating Moore's law, one would estimate that within the next 10-20 years we will reach the atomic size for storing a single bit of information. At that point, quantum effects will become unavoidably dominant.

It is then natural to raise the following question: can one exploit quantum mechanics to transmit and process information? The aim here is to build a quantum computer based on quantum logic, that is, it processes the information and performs logic operations in agreement with the laws of quantum mechanics. Quantum computers were envisioned by Feynman in the 1982 (Hey 2002) and more recently quantum information processing and communication has established itself as one of the hot topic fields in contemporary science. The great challenge is to build quantum machines based on quantum logic, which process the information and perform logic operations by exploiting the laws of quantum mechanics. The power of quantum computation is rooted in typical quantum phenomena, such as the superposition of quantum states

and entanglement. There is an inherent quantum parallelism associated with the superposition principle. In simple terms, a quantum computer can process a large number of classical inputs in a single run. On the other hand, this implies a large number of possible outputs. It is the task of quantum algorithms, which are based on quantum logic, to exploit the inherent quantum parallelism of quantum mechanics to highlight the desired output. Therefore, besides the imposing technological challenges, to be useful quantum computers also require the development of appropriate quantum software, that is, of efficient quantum algorithms.

The purpose of these short notes is to give a flavor of the state of the art of quantum computation, focusing on open problems and perspectives rarely discussed in the literature. Only a few papers useful as entry points will be quoted. For a more complete bibliography, see the textbooks (Benenti & et al. 2004-2007; Nielsen & Chuang 2000)

2. Quantum computers and quantum mechanics

An operative quantum computer would allow us to perform countless experimental tests on the foundations of quantum mechanics, at present only considered “gedanke experimente”. For instance, tests on the nonseparability of quantum mechanics (Aspect 1976) could be implemented between two entangled registers moving one with respect to the other. In order to test possible gravitational effects on quantum states, one register could be located on the earth’s surface and the other in a satellite.

The basic principles of quantum mechanics required for the working of a quantum computer are very simple and can be conveniently summarized by the quantum circuit drawn in Fig. 1. Let us consider a n -qubit quantum computer, whose input state is described by a density operator ρ acting on the n -qubit Hilbert space of dimension 2^n . The environment is represented by m additional qubits, initially in a (pure) state $|0^m\rangle$. The overall system-environment evolution is described by the unitary matrix U , which includes both the quantum gates and the undesired system-environment interaction inducing decoherence. Finally, the environment is disregarded (mathematically, we trace the overall state over the environmental degrees of freedom) and n'' qubits are measured in the computational basis (say, the basis of the eigenstates $|0\rangle$ and $|1\rangle$ of the Pauli spin operator σ_z). This measurement induces the wave function collapse and we end up with the state ρ' for the remaining $n' = n - n''$ qubits.

It should be stressed that the theoretical picture summarized in Fig. 1 only refers to quantum systems with finite-dimensional Hilbert spaces. Furthermore, one should remember that there is an existing tension between quantum mechanics and the relativistic theory of gravitation, still not solved by quantum gravity theories such as loop quantum gravity and

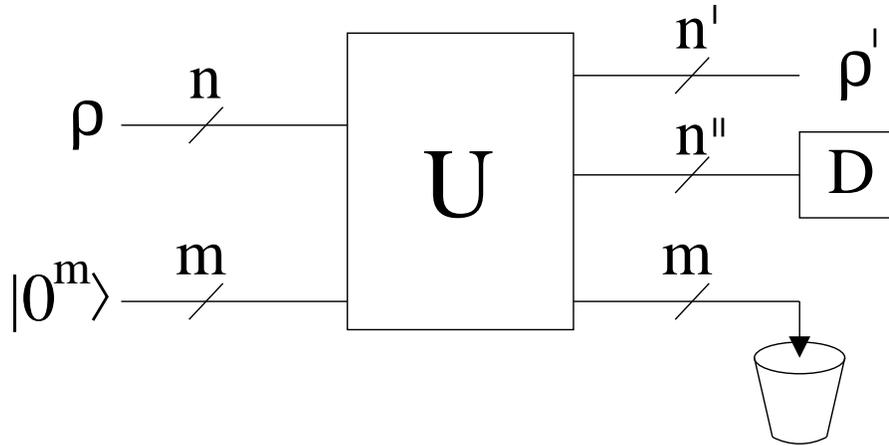


Fig. 1.— A quantum circuit summarizing the working of a quantum computer.

superstrings theories. Of course, one could assume quantum gravity effects to be important only at length scales below the Planck scale (approximately 10^{-33} cm). On the other hand, given the absence of a complete theory, it cannot be excluded that effects would appear at a larger length scale, possibly hampering the construction of a quantum computer with many qubits. We emphasize that a quantum computer useful for the solution of problems in quantum chemistry or for factoring integers beyond the capabilities of a classical computer would require $10^2 - 10^4$ qubits. On the other hand, the predictions of quantum mechanics have been so far tested with up to 11 significant digits (we refer here to the experiments measuring the magnetic moment of the electron). As the size of the Hilbert space grows exponentially with the number of qubits, the implementation of a quantum computer with $10^2 - 10^4$ qubits would represent a verification of the validity of quantum mechanics much beyond the tests so far realized.

At any rate, even the first, few-qubit demonstrative experiments are remarkable, not only for quantum computation but also for addressing fundamental questions on quantum mechanics, such as the nature of the frontier between quantum and classical worlds or the nature of quantum entanglement in complex many-body systems. We can gain from quantum computation and information a better intuition of the weird world of quantum mechanics and of its most counterintuitive aspects. It is here useful to remind the reader that even Schrödinger wrote (Schrödinger 1952) that “*We never experiment with just one electron or atom or (small) molecule. In thought-experiments we sometimes assume that we do; this invariably entails ridiculous consequences ... we are not experimenting with single particles, any more than we can raise Ichthyosauria in the zoo*”. It is absolutely remarkable that only fifty years later experiments on single electrons, atoms and molecules are routinely performed in laboratories all over the world, for instance in few-qubit implementations of quantum algorithms.

3. The problem of decoherence

In practice, any quantum system is open; namely, it is never perfectly isolated from the environment. The word decoherence, used in its broader meaning, denotes any quantum noise process due to the unavoidable coupling of the system to the environment. Decoherence can be considered as the ultimate threat to the actual implementation of any quantum computation or communication protocol. Indeed, decoherence invalidates the quantum superposition principle, which lies at the heart of the potential power of any quantum algorithm. On the other hand, any quantum information protocol must end up with a measurement, converting quantum states into classical outcomes, and decoherence plays a key role in this quantum measurement process. The decoherence problem is very complex, both theoretically and experimentally. To give an idea of its complexity it is sufficient to remind the reader that in the most general evolution of a n -qubit quantum computer (see Fig. 1) the action of the environment is described by $n^4 - n^2$ real parameters; that is, 12 parameters for a single qubit, 240 for two-qubits, and so on. The main problem is that it appears difficult to attach a concrete physical meaning to all these parameters for more than one qubit. In the simplest case, $n = 1$, the 12 parameters are associated to rotations, deformations and displacements of the single-qubit Bloch sphere. On the other hand, even in this simple case quantum noise is usually described by means of phenomenological theories involving a smaller number of parameters, for instance just the dephasing and relaxation time scales. It is indeed very difficult, even for a single qubit, to perform experimentally a complete “tomography” of noise and so determine all the above 12 parameters.

According to the standard theory of quantum mechanics decoherence is a technical, though very difficult problem. In other words, there are no fundamental limitations to the generation and persistence of arbitrarily complex entangled states. However, it is worth mentioning that there exist interpretations of quantum mechanics in which “intrinsic decoherence” for isolated systems of particles is introduced, see for instance Ghirardi et al. (1986). In general, the problem of decoherence is strictly connected to the emergence of classicality in a world governed by the laws of quantum mechanics. Even though this latter problem has fascinated scientists since the dawn of quantum mechanics, there exists as yet no theory solving it in a fully satisfactory way.

4. Quantum error correction

If errors are not corrected, the error per gate that can be tolerated tends to zero when the number of gates tends to infinity. It is then clear that, to allow arbitrarily long computations with reasonable physical resources, error correction is necessary. The question is how to protect

quantum information from errors. It is indeed evident that coherent superposition of quantum states are very fragile and prone to decoherence. However, quantum error-correcting codes fighting the effects of noise have been developed. These codes are very efficient for the correction of single-qubit errors, acting independently on each qubit. On the other hand, when several qubits couple identically to the environment (collective decoherence), it is possible to encode the information in decoherence-free subspaces. However, it is clear from the above considerations on decoherence that the problem is in general complex. For instance, when dealing with two-qubit errors one has to distinguish between two single-qubit errors (described by 2×12 parameters) and “true” two-qubit errors, described by $240 - 2 \times 12$ parameters. This latter class of errors has not received enough attention in the literature, so far.

5. Experimental implementations

The great challenge of quantum computation is to experimentally realize a quantum computer. We need approximately 30 – 100 qubits and thousands of quantum gates to build a quantum simulator capable of solving quantum mechanical problems beyond the capabilities of present-day supercomputers. On the other hand, thousand of qubits are required to outperform existing classical computers in computational problems such as molecular structure determination. Many requirements must be fulfilled in order to achieve this imposing objective. We require a collection of two-level quantum systems that can be prepared, manipulated and measured at will. That is, our purpose is to be able to control and measure the state of a many-qubit quantum system. A useful quantum computer must be scalable since we need a rather large number of qubits to perform non-trivial computations. In other words, we need the quantum analogue of the integrated circuits of a classical computer. Qubits must interact in a controlled way if we wish to be able to implement a universal set of quantum gates. Furthermore, we must be able to control the evolution of a large number of qubits for the time necessary to perform many quantum gates.

Given the generality of the requirements to build a quantum computer, many physical systems might be good candidates, and very interesting few-qubit experiments have been performed with nuclear magnetic resonance techniques applied to nuclear spins of molecules in the liquid state, with neutral atoms and photons interacting in a resonant cavity, with cold ions in a trap and with solid-state qubits. Qubits made out of solid-state devices may offer great advantages since fabrication by established lithographic methods allows for scalability (at least in principle). Moreover, another important feature of solid-state devices is their flexibility in design and manipulation schemes. Indeed, in contrast to “natural” atoms, “artificial” solid-state atoms can be lithographically designed to have specific characteristics such as a

particular transition frequency. This tunability is an important advantage over natural atoms. Finally, solid-state qubits are easily embedded in electronic circuits and can take advantage of the rapid technological progress in solid-state devices as well as of continuous progress in the field of nanostructures. On the other hand, it should be remarked that there is a great variety of decoherence mechanisms, still not well understood, in solid-state devices. When the problem of decoherence is taken into account for a complex many-qubit system, which we require to perform coherent controlled evolution, then large-scale quantum computers appear unrealistic with present technology. On the other hand, we should bear in mind that also the technological development of the classical computer took decades and that breakthroughs (such as the transistor was for the classical computer) are always possible.

Quantum technologies are not limited to quantum computation but include other relevant applications such as quantum cryptography. It appears probable that in the near future quantum cryptography will be the first quantum-information protocol to find commercial applications, and indeed quantum cryptosystems are already sold. Here the question is how extensive the market will be and this will largely depend on the transmission rates, at present limited to the kHz range. The development of fast single-photon sources and high-efficiency detectors is required to improve significantly the transmission rates, thus broadening the prospects of quantum cryptography.

6. Are quantum computers the ultimate frontier?

It is worth mentioning that some years ago computers using electronic and optical processes between molecules were envisaged (see Hameroff (1987) and Carter (1982)) and presented as ultimate computers (note that these ideas have also stimulated a very interesting experimental activity). However, it should be stressed that molecular computers are based on classical Boolean logic. In contrast, quantum computers replace the laws of classical physics applied to computation with the more fundamental laws of quantum mechanics. Computers are physical devices, whose working is governed by physical laws, so the question whether quantum computers are the ultimate frontier is strictly related to the question whether quantum mechanics is the theory describing all physical phenomena at all scales.

If we remain in the framework of the quantum systems with finite-dimensional Hilbert spaces, it is possible to simulate a quantum computer by means of a classical one. Therefore, the class of all functions computable by a quantum computer is equivalent to the class of all functions computable by a classical Turing machine. Of course, there might be for certain problems an exponential gain in time and/or memory resources. This means that a large-scale quantum computer could solve problems in practice beyond the reach of classical Turing

machines or, in general, of classical computers based on classical physics. Indeed, it is clearly impossible to implement a Turing machine with a tape divided into 10^{80} cells, this number being comparable to the number of nucleons in our universe. On the other hand, a quantum register of 300 qubits would overcome this limit.

At present, there are no experimental results in contradiction with the postulates of quantum mechanics. However, we cannot exclude that in the future new experiments could require modifications of the foundations of quantum mechanics, perhaps supporting more powerful computers. It is therefore worth mentioning some results beyond “standard” quantum computation:

- If the superposition principles of quantum mechanics might be violated, that is, if the time evolution of quantum states might be nonlinear, then quantum computers could be used to solve **NP**-complete problems in polynomial time (Abrams & Lloyd 1998). The computational class **NP** is the class of problems whose solution can be verified in polynomial time. A problem in the class **NP** is **NP**-complete if any problem in **NP** is reducible to it by means of a mapping involving only polynomial resources. We stress that no known polynomial time algorithms for the solution of **NP**-complete problems exist, both on a classical and on a quantum computer based on standard, linear quantum mechanics.
- Kieu’s algorithm (Kieu 2003). It has been claimed that, using infinite-dimensional Hilbert spaces with special spectral properties of the Hamiltonian operator, it is possible to solve the Hilbert’s tenth problem (Hilbert 1902). Note that this problem is equivalent to the Turing halting problem and therefore noncomputable on a classical as well as on a standard quantum computer.

7. Final remarks

In order to evaluate the future impact of quantum computation, the main question under discussion is: is it possible to build a useful quantum computer that could outperform existing classical computers in important computational tasks? And, if so, when? The difficulties are huge. Besides the problem of decoherence, we should also remark on the difficulty of finding new and efficient quantum algorithms. We know that problems such as molecular structure determination can be solved efficiently on a quantum computer, but we do not know the answer to the following fundamental question: What class of problems could be simulated efficiently on a quantum computer? Quantum computers open up fascinating prospects, but it does not seem likely that they will become a reality with practical applications in a few years. How

long might it take to develop the required technology? Even though unexpected technological breakthroughs are, in principle, always possible, one should remember the enormous effort that was necessary in order to develop the technology of classical computers.

We can certainly say that the computational power at our disposal increased enormously over the years and the computer hardware changed from pebbles to mechanical and electromechanical computers, vacuum tubes, up to transistors and integrated circuits. The more optimistic predictions always underestimated the development of computers and we might hope the same will happen for quantum computers. Of course, the time when a quantum computer will be on the desk in our office is uncertain. In any event, what is certain is that we are witnessing the emergence of a very promising field of investigation in physics, mathematics and computer science.

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